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Higgs-mass predictions and electroweak precision observables in the Standard Model and the MSSM*

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1. INTRODUCTION

By confronting the electroweak precision data with the theory, i.e. the electroweak Standard Model (SM) or extensions of it, most prominently the Minimal Supersymmetric Standard Model (MSSM), it is possible to test the theory at its quantum level, where all parameters of the model enter. Within the SM, a fit to the precision data allows to constrain the mass of the Higgs boson, being the last missing ingredient of the model. The present 95% C.L. upper bound on the Higgs-boson mass is given by $M_{\rm H} < 188$ GeV [1]. The dependence of the precision observables on $M_{\rm H}$ is only logarithmic in leading order, while they depend quadratically on the top-quark mass [2]. Thus, in order to derive indirect constraints on the Higgs-boson mass a very high precision of the experimental data and the theoretical predictions is needed.

While within the SM only the mass of the Higgs boson has not been experimentally determined so far, the MSSM in its unconstrained form (i.e. without specific assumptions about the SUSY-breaking mechanism) introduces more than 100 unknown free parameters (masses, mixing angles, etc.). A precise determination of the model parameters will not only be important in order to investigate whether the MSSM is consistent with the data, but also to infer possible patterns of the underlying SUSY-breaking mechanism. In this context the indirect constraints on the model from precision data will often be complementary to the information from the direct production of SUSY particles. Furthermore, a very stringent direct test of the MSSM is possible. In contrast to the SM, the mass of the lightest $\mathcal{CP}$-even Higgs boson, $m_{\rm h}$, is not a free parameter in the MSSM but is calculable from other parameters of the model. This results in the tree-level bound $m_{\rm h} < M_{\rm Z}$, which however is strongly affected by large radiative corrections [3], shifting it to about $m_{\rm h} \lesssim 135$ GeV at the two-loop level [4, 5, 6, 7, 8]. If the lightest $\mathcal{CP}$-even Higgs boson of the MSSM will be detected, its mass will play an important role as a precision observable. The prospective accuracy at the LHC is $\Delta m_{\rm h} \approx 0.2$ GeV [9]. At a future linear collider this could be improved to $\Delta m_{\rm h} \approx 0.05$ GeV [10], while at a future muon collider even an accuracy of $\Delta m_{\rm h} \approx 0.1$ MeV [11] could be achievable.

2. PRECISION OBSERVABLES IN THE SM AT PRESENT AND FUTURE COLLIDERS

The present bound on the Higgs-boson mass derived within the SM by comparing the experimental data with the theoretical predictions arises mainly from the precision measurements of the mass of the W boson, $M_{\rm W} = 80.419 \pm 0.038$ GeV [1], and the effective leptonic weak

mixing angle at the Z-boson resonance, $\sin^2\theta_{\mathrm{eff}} = 0.23149 \pm 0.00017$ [1]. Theoretical uncertainties in the predictions for the observables arise both from unknown higher-order corrections and from the experimental errors of the input parameters (in particular m_{t}, $\Delta\alpha_{\mathrm{had}}$) used for the theoretical predictions.

The constraints on M_{H} arising from the prediction for the W-boson mass are illustrated in Fig. 1, where the most recent SM result for M_{W} [12] is compared with the current experimental value. The present 95% C.L. lower bound on M_{H} from the direct search of $M_{\mathrm{H}} = 107.9$ GeV [13] is also indicated. The plot shows the well-known preference for a light Higgs boson within the SM. Confronting the theoretical prediction (allowing a variation of m_{t}, which at present dominates the theoretical uncertainty, within 1σ) with the 1σ region of $M_{\mathrm{W}}^{\mathrm{exp}}$ and the 95% C.L. lower bound on M_{H}, only a rather small region in the plot matches all three constraints.

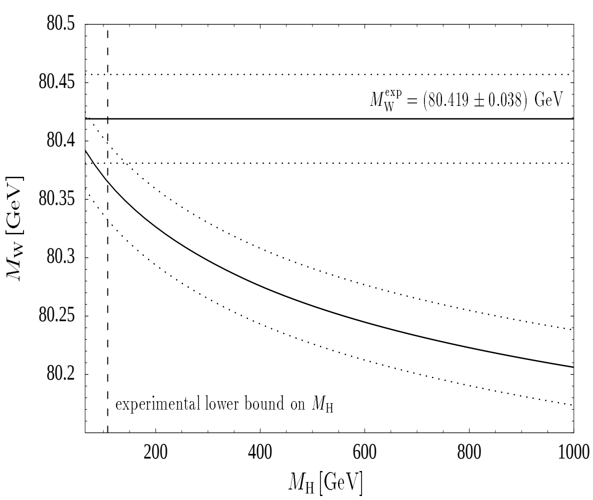

Figure 1. The prediction for M_{W} as a function of M_{H} for $m_{\mathrm{t}} = 174.3 \pm 5.1$ GeV is compared with the current experimental value, $M_{\mathrm{W}}^{\mathrm{exp}} = 80.419 \pm 0.038$ GeV [1], and the experimental 95% C.L. lower bound on the Higgs-boson mass, $M_{\mathrm{H}} = 107.9$ GeV [13].

At the next generation of colliders a significant improvement can be expected in the experimental accuracies of both the observables employed for testing the theory and of the input parameters used for deriving the theoretical predictions. This is illustrated in Fig. 2, where the SM predictions for M_{W} and $\sin^2\theta_{\mathrm{eff}}$ are compared with the experimental accuracy (assuming the present experimental central values of the observables) obtainable at LEP2, SLC and the Tevatron (Run IIA) as well as with prospective future accuracies at the LHC and at a high-luminosity linear collider in a dedicated low-energy run (GigaZ). The experimental accuracies assumed in Fig. 2 for LEP2/Tevatron, LHC and GigaZ are $\Delta M_{\mathrm{W}} =$ 30 MeV, 15 MeV, 6 MeV and $\Delta\sin^2\theta_{\mathrm{eff}} = 1.7\times10^{-4}$, 1.7×10^{-4}, 1×10^{-5}, respectively. The allowed region of the present SM prediction corresponds to varying M_{H} in the interval 90 GeV $\leq M_{\mathrm{H}} \leq$ 400 GeV and m_{t} within its current experimental 1σ uncertainty. The theoretical prediction at GigaZ, assuming that the Higgs boson has been found, is shown for three values of the Higgs-boson mass, $M_{\mathrm{H}} = 120, 150, 180$ GeV, and an uncertainty of $\delta m_{\mathrm{t}} = \pm 200$ MeV and $\delta\Delta\alpha = \pm 7\times10^{-5}$ is taken into account in this case. As can be seen in Fig. 2, the precision observables M_{W} and $\sin^2\theta_{\mathrm{eff}}$ provide a very sensitive test of the theory, in particular in the case of the GigaZ accuracy [14, 15].

This fact manifests itself in the precision with which M_{H} can indirectly be determined within the SM. While at present the 1σ uncertainty of $\delta M_{\mathrm{H}}/M_{\mathrm{H}}$ derived from all data amounts to more than ±60%, at GigaZ an accuracy of about $\delta M_{\mathrm{H}}/M_{\mathrm{H}} = \pm 7\%$ can be expected [15],[1] which is of the same order of magnitude as presently the indirect determination of m_{t} from the precision data (without using its experimental value).

3. CONSTRAINTS ON $\tan\beta$ FROM THE HIGGS SEARCH AT LEP

The upper bound of about $m_{\mathrm{h}} \lesssim 135$ GeV on the mass of the lightest $\mathcal{CP}$-even Higgs boson is a definite and fairly robust prediction of the MSSM,

[1]For the future theoretical uncertainties from unknown higher-order corrections (including the uncertainties from $\delta\Delta\alpha$) we have assumed $\delta M_{\mathrm{W}}(\text{theory}) = \pm 3$ MeV and $\delta\sin^2\theta_{\mathrm{eff}}(\text{theory}) = \pm 3\times10^{-5}$ [15].

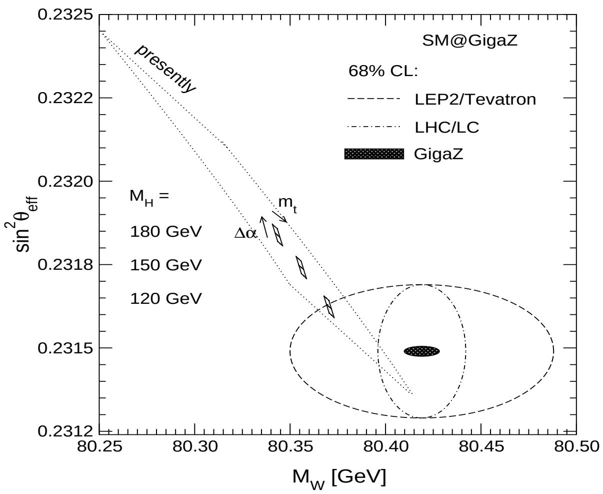

Figure 2. The present and prospective future theoretical predictions in the SM (for three hypothetical values of the Higgs-boson mass) are compared with the experimental accuracies at LEP2/Tevatron (Run IIA), the LHC and GigaZ.

which can be tested at the present and the next generation of colliders. By comparing the experimental limit on m_h from the search at LEP2 with the theoretical result for the upper bound on m_h in the MSSM as a function of $\tan\beta$ (the ratio of the vacuum expectation values of the two Higgs doublets) it is possible to derive constraints on $\tan\beta$.

The theoretical input used in the analysis of the search at LEP for the lightest $\mathcal{CP}$-even Higgs boson of the MSSM has recently been improved in two ways. Firstly, a new "benchmark scenario" has been proposed [16, 17] which slightly generalizes the parameter settings (for $m_\mathrm{t} = 174.3$ GeV and $M_\mathrm{SUSY} = 1$ TeV kept fixed) used so far in the LEP analyses. It leads to a more conservative upper bound on m_h as a function of $\tan\beta$ ("$m_\mathrm{h}^\mathrm{max}$ scenario"). Secondly, a new Feynman-diagrammatic two-loop result for m_h became available [7] which incorporates genuine two-loop contributions (see Ref. [18]) that were not contained in the renormalization-group (RG) improved one-loop effective potential result [5] previously used for the data analyses. This is illustrated in Fig. 3, where the upper bound on m_h is shown as a function of $\tan\beta$. The difference between the dashed and the dotted curve is the effect of changing the previously used parameter settings to the new LEP benchmark values ($m_\mathrm{h}^\mathrm{max}$ scenario), while the difference between the full curve and the dashed curve is caused by using the Feynman-diagrammatic (FD) result instead of the previous result (RG). As can be seen in the figure, both changes (for $\tan\beta \gtrsim 1$) result in a sizable shift towards higher values of m_h. Their combined effect amounts to an upward shift of m_h of up to almost 10 GeV.

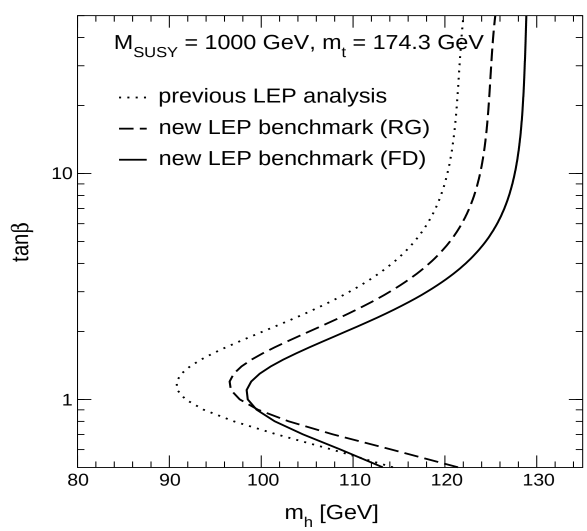

Figure 3. The mass of the lightest $\mathcal{CP}$-even Higgs boson in the MSSM is shown as a function of $\tan\beta$ using the RG result with the parameter settings previously used for the LEP analyses, the RG result in the $m_\mathrm{h}^\mathrm{max}$ scenario, and the FD result in the $m_\mathrm{h}^\mathrm{max}$ scenario (see text).

This upward shift in $m_\mathrm{h}(\tan\beta)$ gives rise to the fact that the excluded $\tan\beta$ region in the $m_\mathrm{h}^\mathrm{max}$ scenario using the combined data taken by the four LEP collaborations up to the end of 1999 is smaller than in the previous LEP analyses, despite the fact that the experimental lower bound on m_h as function of $\tan\beta$ has considerably improved. In Fig. 4, the excluded region resulting from the combination of the data of the

four LEP experiments is compared with the upper (and lower) bound within the MSSM (marked as "theoretically inaccessible") obtained with the program *FeynHiggs* [19] based on the Feynman-diagrammatic result. The upper plot shows the case of the $m_{\rm h}^{\rm max}$ scenario, for which an excluded region of $0.7 < \tan\beta < 1.8$ can be inferred [13]. The lower plot shows the so-called no-mixing scenario [17], where vanishing mixing in the scalar top sector is assumed (the other parameters are the same as in the $m_{\rm h}^{\rm max}$ scenario), resulting in significantly smaller values of $m_{\rm h}$. In this scenario an excluded region of $0.4 < \tan\beta < 4.1$ is obtained [13].

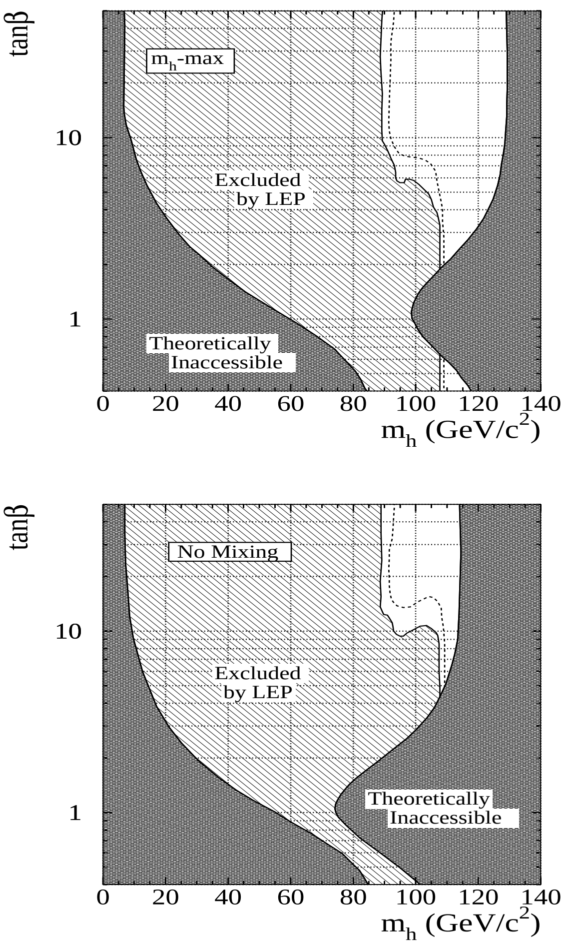

Figure 4. The 95% C.L. bounds on $m_{\rm h}$ in the $m_{\rm h}^{\rm max}$ and the no-mixing scenario obtained from combining the data of the four LEP experiments at 192 to 202 GeV with earlier data taken at lower energies are compared with the upper bound on $m_{\rm h}$ within the MSSM [13]. The full lines represent the actual observation, while the dashed lines are the expected limits based on "background only" Monte Carlo simulations.

4. PRECISION TESTS OF THE MSSM AT GigaZ

Similarly to the case of the SM, the predictions for $M_{\rm W}$ and $\sin^2\theta_{\rm eff}$ can also be employed for an indirect test of the MSSM. We consider here the unconstrained MSSM with real parameters. For the precision observables and $m_{\rm h}$ mainly the parameters of the scalar top and bottom sector and of the Higgs sector are relevant. In Fig. 5 the predictions within the SM and the MSSM in the $M_{\rm W}$–$\sin^2\theta_{\rm eff}$ plane are compared with the experimental accuracies at LEP2/Tevatron, the LHC (and an LC without the GigaZ mode) and GigaZ. For the SUSY contributions to $M_{\rm W}$ and $\sin^2\theta_{\rm eff}$ we use the complete one-loop results in the MSSM [20] combined with the leading higher-order QCD corrections [21].[2] The allowed region of the SM prediction is the same as in Fig. 2, while in the MSSM prediction besides the uncertainty of $m_{\rm t}$ also the SUSY parameters are varied. Fig. 5 shows that the predictions of the two models have only a relatively small overlap in the $M_{\rm W}$–$\sin^2\theta_{\rm eff}$ plane (corresponding to the SM with a light Higgs boson and the MSSM in the decoupling region). With the GigaZ accuracy, the precision observables provide a high sensitivity to deviations from both models.

The constraints from the precision data on $M_{\rm W}$, $\sin^2\theta_{\rm eff}$, etc. at GigaZ can be combined with those from a prospective measurement of $m_{\rm h}$ with a precision of $\delta m_{\rm h} = 0.05$ GeV. This could allow to indirectly probe the masses of particles in supersymmetric theories that might not be accessible directly neither at the LHC nor at the LC.

As an example, we consider the case where at the LHC and the LC the mass of the lighter scalar

[2]The recent electroweak two-loop results of the SM part in the MSSM [12, 22] have not been taken into account, since no genuine MSSM counterpart is available so far.

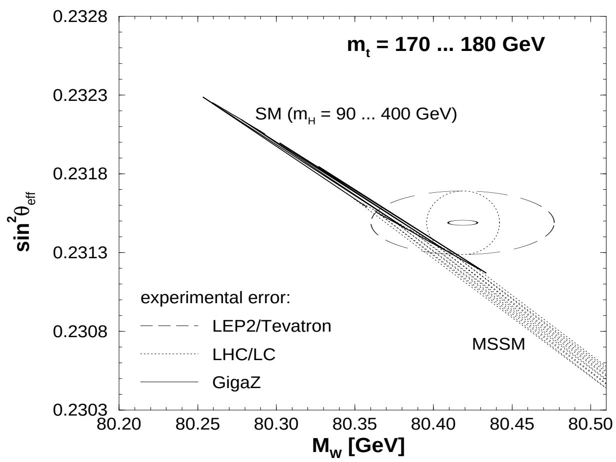

Figure 5. Theoretical predictions of the SM and the MSSM in the $M_{\rm W}$–$\sin^2\theta_{\rm eff}$ plane compared with expected experimental accuracies at LEP2/Tevatron, the LHC and GigaZ.

top quark, $m_{\tilde{t}_1}$, and the $\tilde{t}$-mixing angle, $\theta_{\tilde{t}}$, are measurable very well. On the other hand, we assume no experimental information on the heavier $\tilde{t}$-particle, $\tilde{t}_2$, and on the mass of the $\mathcal{CP}$-odd Higgs-boson, $M_{\rm A}$. For large masses, $\tilde{t}_2$ and the heavy Higgs bosons A, H and $\rm H^\pm$ are very difficult to observe as a consequence of background problems at the LHC and lacking energy at the LC. For $\tan\beta$ we assume that from measurements in the gaugino sector either a fairly tight bound can be set if $\tan\beta$ turns out to be relatively small, e.g. $2.5 \leq \tan\beta \leq 3.5$, or otherwise only a lower bound, e.g. $\tan\beta \geq 10$ (see e.g. Ref. [23]). As for the other parameters, the following values are assumed, with uncertainties as expected from LHC [9] and TESLA [10]: $m_{\tilde{t}_1} = 500 \pm 2$ GeV, $\sin\theta_{\tilde{t}} = -0.69 \pm 2\%$, $A_{\rm b} = A_{\rm t} \pm 10\%$, $\mu = -200 \pm 1$ GeV, $M_2 = 400 \pm 2$ GeV and $m_{\tilde{g}} = 500 \pm 10$ GeV. In our analysis we assume a future uncertainty in the theoretical prediction of $m_{\rm h}$ of ± 0.5 GeV.

In this scenario, assuming as experimental values $M_{\rm W} = 80.400 \pm 0.006$ GeV, $\sin^2\theta_{\rm eff} = 0.23140 \pm 0.00001$ and $m_{\rm h} = 115.00 \pm 0.05$ GeV, we find the allowed region in the $M_{\rm A}$–$m_{\tilde{t}_2}$ plane shown in Fig. 6 [15]. In the case of large $\tan\beta$, where the prediction for $m_{\rm h}$ depends only mildly on $\tan\beta$, the constraint from the $m_{\rm h}$ measurement yields a rather tight bound on $m_{\tilde{t}_2}$ [24]. In combination with the information from the data on $M_{\rm W}$ and $\sin^2\theta_{\rm eff}$ this gives rise to a relatively small allowed region in the $M_{\rm A}$–$m_{\tilde{t}_2}$ plane (660 GeV $\lesssim m_{\tilde{t}_2} \lesssim$ 680 GeV, 200 GeV $\lesssim M_{\rm A} \lesssim$ 800 GeV). In the small $\tan\beta$ region, on the other hand, where $m_{\rm h}$ depends very sensitively on $\tan\beta$, a larger allowed region in the $M_{\rm A}$–$m_{\tilde{t}_2}$ plane is obtained. In both cases, however, an upper bound on $M_{\rm A}$ can be established, which mainly arises from the precise measurement of $\sin^2\theta_{\rm eff}$. Note that we have assumed a measured value of $\sin^2\theta_{\rm eff}$ in accordance with the MSSM prediction in a region where the SUSY particles do not completely decouple, i.e. a value slightly below the corresponding SM prediction (see Fig. 5). For the experimental accuracy at an LC without the GigaZ mode no bound on $M_{\rm A}$ could be inferred.

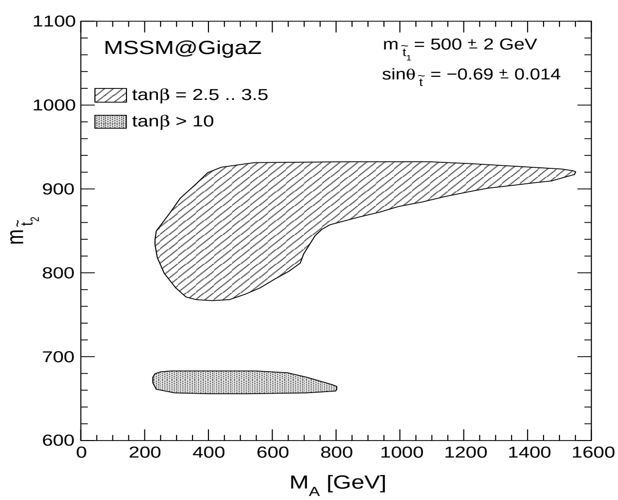

Figure 6. The region in the $M_{\rm A}$–$m_{\tilde{t}_2}$ plane, allowed by $1\,\sigma$ errors obtained from the GigaZ measurements of $M_{\rm W}$ and $\sin^2\theta_{\rm eff}$, taking as hypothetical values $M_{\rm W} = 80.400 \pm 0.006$ GeV, $\sin^2\theta_{\rm eff} = 0.23140 \pm 0.00001$ and $m_{\rm h} = 115.00 \pm 0.05$ GeV. $\tan\beta$ is assumed to be experimentally constrained by $2.5 \leq \tan\beta \leq 3.5$ or $\tan\beta \geq 10$.

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